Math 201 - Exam 2 (Fall 08)

T. Tlas

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- Please answer questions 1, 2 and 4 on the same sheet of paper on which it is written (after the line following the question). Any part of your answer written on the wrong page will not be graded. Question 3 has an extra sheet of paper on which you can write your answer.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 4 problems in total. All questions have several parts to them. Make sure that you attempt them all.

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Name :

ID # :

Section Number :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$TOTAL$	

Problem 1

At a point P a function of two variables $f(x, y)$ increases most rapidly in the direction of $\vec{\mathbf{i}} + \vec{\mathbf{j}}$.

i- (5 points) If

$$\frac{\partial f}{\partial x}(P) = 1$$

find $\vec{\nabla} f(P)$

ii- (5 points) Find the equation of the line tangent to the level curve of f passing through P.

iii- (15 points) Find the rates of change of f in the directions of $\vec{\mathbf{i}} + 2\vec{\mathbf{j}}$ and $\vec{\mathbf{i}} - \vec{\mathbf{j}}$.

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Problem 2

(9 points each) In each of the following three cases, either show that the limit of the function as $(x, y) \to (0, 0)$ does not exist, or if it does find the limit. Also, when possible, extend to (0,0) the definition of the function by continuity.

i-

$$f(x,y) = \frac{y^2\, x}{y^4 + x^2}$$

ii-

$$g(x,y) = \frac{|y|^{\frac{3}{2}}\, x}{|y|^3 + x^2}$$

iii-

$$h(x,y) = \frac{|y|^{\frac{3}{2}}\, x}{y^2 + x^2}$$

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Problem 3

Consider the function of two independent variables

$$f(x, y) = x^4 + y^4 + 2\,x^2y^2 - 2\,x^2 + 2\,y^2 + 1$$

i- (10 points) What are the domain and range of the function? Is the domain closed? open? bounded? Give brief justifications to your answers.

ii- (15 points) Draw the level curve passing through $(\sqrt{2}, 0)$.

iii- (5 points) Find the tangent line (or lines, if there is more than one) to the level curve from part (ii) at (0,0).

iv- (10 points) Find the maximum value of the function $g(x, y) = y^2 - x^2$ when it is restricted to the curve in part (ii).

Hint: You might find polar coordinates useful for some parts of this question.

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ADDITIONAL SHEET FOR PROBLEM 3 ANSWER

Problem 4

i- (4 points) Prove that if O_1 and O_2 are two open sets in the plane, then

$$O = O_1 \cup O_2$$

is also open. In other words, show that the union of two open sets is still an open set.

ii- (4 points) Prove that if $\{O_n\}_{n=1}^{\infty}$ is a collection of open sets in the plane then

$$O = \bigcup_{n=1}^{\infty} O_n$$

is also open. In other words, show that the union of infinitely many open sets is still an open set.

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